Specimen Paper A

1 Express in partial fractions $\dfrac{2x}{(x+3)^2}$ **3**

2 Differentiate (a) $y=6^x$ (b) $y = 2xe^{\sec 2x}$ **3,3**

3 Find the equation of the locus of $|z-2i|=3$, z a complex number.
Describe geometrically the locus. **4**

4 Use integration by parts twice to find $\int x^2 e^{-x}\,dx$ **5**

5 Prove by induction $\sum_{r=1}^{n}(2r+1) = n(n+2)$, $n \geq 1$, n a whole number. **4**

6 A curve is defined by the equation $x^2 - 3xy + y = 5$.
Find the equation of the tangent at the point where x=-1. **5**

7 Use the substitution $x = 2\sin\theta$ to evaluate $\int_0^1 \dfrac{2}{\sqrt{4-x^2}}\,dx$ **5**

8 The sum to infinity of both Geometric series $x + x^2 + x^3 + \ldots$ and
$3 + \dfrac{3(x-1)}{x} + \dfrac{3(x-1)^2}{x^2} + \ldots.$ exist and are both equal provided $\dfrac{1}{2} < x < 1$.

Find the value of x. **5**

9/over

9 For $n \geq 1$, n a natural number prove whether the following Statements are true or false, if false provide a counter example.

(a) $n^3 + n$ is always even.

(b) $n^2 + n + 5$ is always prime **2,1**

10 Find $\underline{a} \times \underline{b}$, $\underline{a} = i + j + 3k$, $\underline{b} = -j + 2k$ **3**

11 Find the Maclaurin series up to x^3 for $f(x) = x\cos x$ **5**

12 **(a)** Find the acute angle between the planes
$2x + y - z = 3$ and $x + z = 0$

(b) Find the point of intersection of the line
$x = 2t + 1,\ y = 3t,\ z = 4$ and $x + z = 0$ **3,3**

13 For a matrix A, $A^3 - A = I$

(a) Find A^{-1} in the form $pA^2 + q$, stating the values of p and q.

(b) Find A^5 in the form $rA^2 + sA + I$, stating the values of r and s.
2,3

14 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 32e^x$$

and given $y = \frac{dy}{dx} = 0$, when x=0 find the particular solution, **7,3**

/over

15 X is the percentage of a class of 15 students who hope to pass a 3 hour Mathematics Exam after attempting T practice papers.

At the first attempt 50% of the class passed.
The attempts are modelled by the differential equation

$$\frac{dX}{dT} = \frac{X(1-X)}{(3+X)}, \quad 0.5 \le X < 1$$

(a) Show that $T = A\ln_e \frac{X^3}{(1-X)^4}$, stating the value A to 2 decimal places.

(b) How many practice papers are needed for 70% of the class to pass ?

(c) The class teacher estimates she will have to produce 9 papers in order that 80 % of the class pass.
Is this estimate reasonable or just *pi* in the sky?

(d) If 12 papers are produced will the % pass rate improve to 90%? **6,2,2,1**

16 Let z be the complex number $z = \cos\theta + i\sin\theta$

(a) Show that $\overline{z} = \frac{1}{z}$ and express $z - \overline{z}$ in terms of $\sin\theta$.
Hence show that $(z-\overline{z})^2 = p\sin^2\theta$, stating the value of p. **3,2**

(b) Using de Moivre's theorem write down an expression for z^2 and hence find an expression for $\frac{1}{z^2}$ **3**

(c) By expanding $(z-\overline{z})^2$ and using the results of (a) and (b) show that $\sin^2\theta = q\cos 2\theta + r$ stating the values of q and r **2**

17 A function f is defined by the formula $f(x) = x + 2 + \frac{4}{x^2}, \; x \neq 0$

(a) Write down the equations of both asymptotes. **2**

(b) Show that f has only one critical point.
Find the coordinates of the point and justify its nature. **4**

(c) Sketch the graph of $y = f(x)$ showing all the main features **1**

(d) Find the equation of $y = f(-x)$ and hence write down the Coordinates of its critical point and the equation of the Non vertical asymptote. **3**

End of question paper Total = 100 mark